

Exercice 1

$$1) \quad z^2 + 6\sqrt{3}z + 36 = 0$$

$$\Delta = 36 \times 3 - 4 \times 36 = -36 = (6i)^2$$

$$z_1 = \frac{-6\sqrt{3} + 6i}{2} = -3\sqrt{3} + 3i$$

$$z_2 = \frac{-6\sqrt{3} - 6i}{2} = -3\sqrt{3} - 3i$$

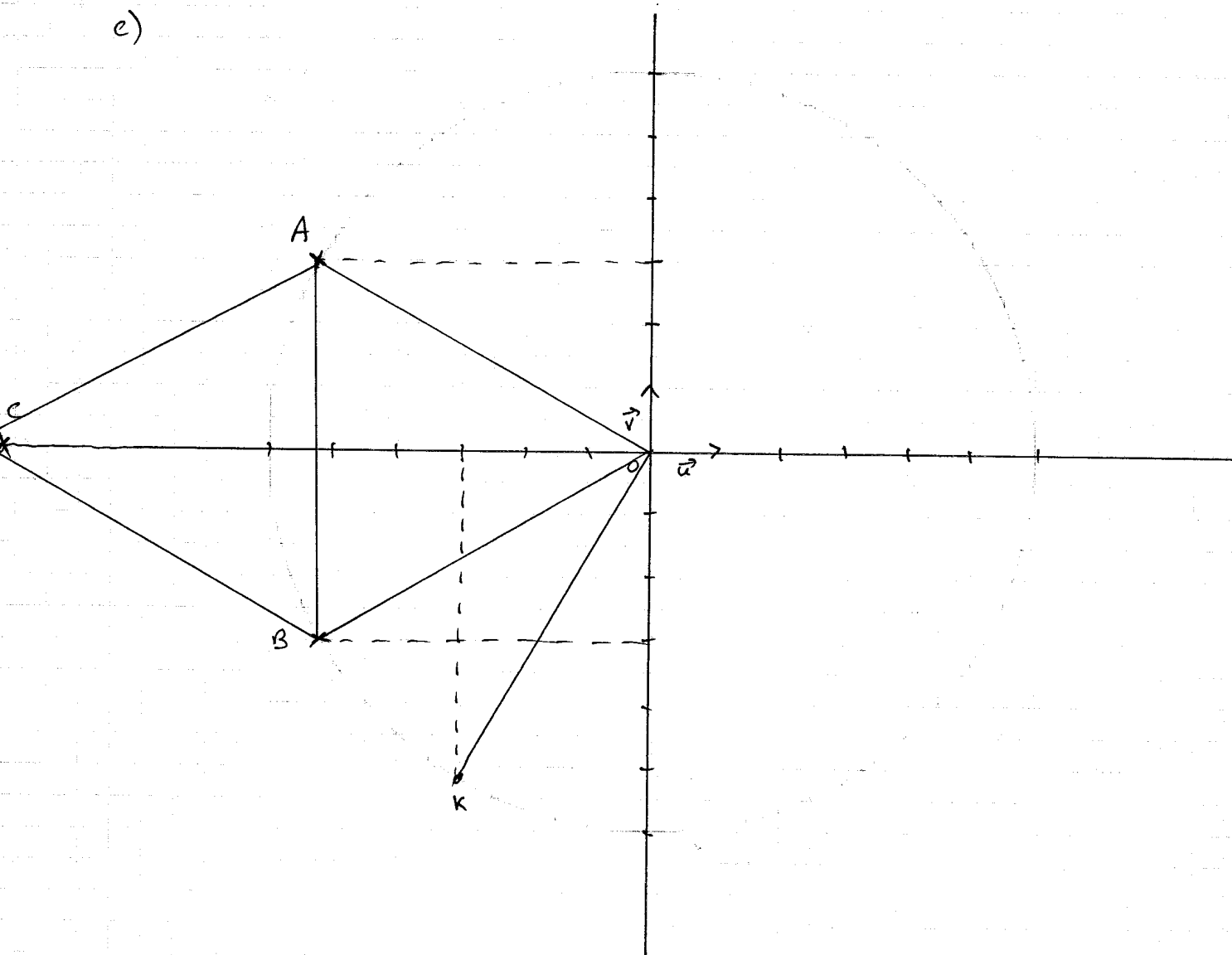
$$2) \quad a) \quad z_A = -3\sqrt{3} + 3i \Rightarrow |z_A| = \sqrt{9 \times 3 + 9} = \sqrt{36} = 6$$

$$\left. \begin{aligned} \Rightarrow \cos \vartheta_A &= \frac{-3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2} \\ \sin \vartheta_A &= \frac{3}{6} = \frac{1}{2} \end{aligned} \right\} \Rightarrow \vartheta_A = \frac{5\pi}{6} \Rightarrow \arg(z_A) = \frac{5\pi}{6}$$

$$z_B = \overline{z_A} \Rightarrow |z_B| = |z_A| = 6 \text{ et } \arg(z_B) = -\arg(z_A) = -\frac{5\pi}{6}$$

$$b) \quad z_A = 6 e^{i \frac{5\pi}{6}}$$

c)



$$3) a) AB = |z_B - z_A| = |-3\sqrt{3} - 3i + 3\sqrt{3} - 3i| = |-6i| = 6$$

$$AC = |z_C - z_A| = |-6\sqrt{3} + 3\sqrt{3} - 3i| = |-3\sqrt{3} - 3i| = \sqrt{9 \times 3 + 9} = 6$$

$$BC = |z_C - z_B| = |-6\sqrt{3} + 3\sqrt{3} + 3i| = |-3\sqrt{3} + 3i| = \sqrt{9 \times 3 + 9} = 6$$

$AB = AC = BC \Rightarrow ABC$  équilatéral.

$$b) |z_A| = OA = 6 \text{ et } |z_B| = OB = 6$$

$$\Rightarrow OA = OB = AC = BC$$

$$\text{de plus } z_C - z_B = -3\sqrt{3} + 3i = z_A \Rightarrow \vec{BC} = \vec{OA}$$

donc  $OABC$  parallélogramme de cotés égaux  $\Rightarrow OABC$  losange.

4)  $OAk$  rectangle isocèle en  $O$

$$a) OA = OK \Rightarrow OK = 6$$

$$OAk \text{ rectangle en } O \Rightarrow (\vec{u}, \vec{OK}) = (\vec{u}, \vec{OA}) + (\vec{OA}, \vec{OK}) = \frac{5\pi}{6} + \frac{\pi}{2} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

ordonnée de  $K$  négative et son abscisse est  $6 \cos \frac{4\pi}{3} = 6 \times (-\frac{1}{2}) = -3$

b)  $(\vec{OA}, \vec{OK}) = \frac{\pi}{2} \Rightarrow K$  image de  $A$  par la rotation de centre  $O$  et d'angle  $\frac{\pi}{2}$

c) écriture complexe de cette rotation:  $z' = e^{i\frac{\pi}{2}} z$

$$\text{donc } z_K = e^{i\frac{\pi}{2}} \times 6 e^{i\frac{5\pi}{6}} = 6 e^{i(\frac{\pi}{2} + \frac{5\pi}{6})} = 6 e^{i\frac{4\pi}{3}}$$

$$\Rightarrow z_K = 6 \left( \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) = 6 \left( -\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = -3 - 3\sqrt{3}i$$