BAC 2008 : STi

Exercice 2

(E) : $y'' + 25y = 0$

1) (E) de la forme $y'' + \omega^2 y = 0$ où $\omega^2 = 25 \Leftrightarrow \omega = 5$

$\Rightarrow$ f solution de $(E) \Leftrightarrow f(x) = A\cos(5x) + B\sin(5x)$

2) f solution de $(E) \Rightarrow f(x) = A\cos(5x) + B\sin(5x)$

$f\left(\frac{\pi}{6}\right) = -2 \Rightarrow A\cos\left(\frac{5\pi}{6}\right) + B\sin\left(\frac{5\pi}{6}\right) = -2 \Leftrightarrow -A\frac{\sqrt{3}}{2} + \frac{B}{2} = -2$

$\Rightarrow -\sqrt{3}A + B = -4$

$f'(x) = -5A\sin(5x) + 5B\cos(5x)$

et $f'(0) = -5 \Rightarrow -5A\underbrace{\sin 0}_{0} + 5B\underbrace{\cos 0}_{1} = -5 \Leftrightarrow 5B = -5 \Leftrightarrow B = -1$

et donc $-\sqrt{3}A - 1 = -4 \Rightarrow -\sqrt{3}A = -3 \Rightarrow A = \frac{-3}{-\sqrt{3}} = \sqrt{3}$

donc $f(x) = \sqrt{3}\cos 5x - \sin 5x$

3) $2\cos\left(5x + \frac{\pi}{6}\right) = 2\left(\cos 5x \cos\frac{\pi}{6} - \sin 5x \sin\frac{5\pi}{6}\right)$

$= 2\left(\frac{\sqrt{3}}{2}\cos 5x - \frac{1}{2}\sin 5x\right) = \sqrt{3}\cos 5x - \sin 5x = f(x)$

4) μ = valeur moyenne de f sur $\left[0; \frac{\pi}{6}\right]$

$\mu = \frac{1}{\frac{\pi}{6} - 0}\int_0^{\pi/6} f(x)\,dx = \frac{6}{\pi}\int_0^{\pi/6} 2\cos\left(5x + \frac{\pi}{6}\right)dx$

$= \frac{12}{\pi}\left[\frac{1}{5}\sin\left(5x + \frac{\pi}{6}\right)\right]_0^{\pi/6} = \frac{12}{\pi} \times \frac{1}{5}\left(\underbrace{\sin\left(\frac{5\pi}{6} + \frac{\pi}{6}\right)}_{=\sin\pi = 0} - \sin\left(\frac{\pi}{6}\right)\right)$

$= \frac{12}{5\pi}\left(-\sin\frac{\pi}{6}\right) = -\frac{1}{2} \times \frac{12}{5\pi} = -\frac{6}{5\pi}$